Immersed Heater-to-Bed Heat Transfer in Liquid-Solid Fluidized Beds

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Published studies of heat transfer in liquid-solid fluidized beds can be classified into two groups. One deals with wall-to-bed heat transfer (Richardson and Mitson, 1958; Richardson and Smith, 1962; Wasmund and Smith, 1967; Brea and Hamilton, 1971; Patel and Simpson, 1977; Kato et al., 1981; Chiu and Ziegler, 1983, 1985; Muroyama et al., 1984, 1986), and the other with immersed heater-to-bed heat transfer (Richardson et al., 1976; Baker et al., 1978; Khan et al., 1983; Kang et al., 1983, 1985; Kim et al., 1986; Kang and Kim, 1987; Magiliotou et al., 1988). Although most of the studies have resulted in correlating equations for the experimental data, few have proposed mechanisms of the heat transfer.

In a study by Wasmund and Smith (1967), the wall thermal resistance was found to increase with the increase in the bed porosity. Their data formed the basis for a series model for wall-to-bed heat transfer in liquid-solid fluidized beds. Richardson et al. (1976) have correlated the immersed heater-to-bed heat transfer data in terms of the j-factor and modified Reynolds number; they have reported that the presence of particles might enhance the heat transfer in liquid-solid fluidized beds because of the tendency of particles to disturb the boundary layer at the heat transfer surface. Patel and Simpson (1977) have suggested that the principal mechanism of heat transfer in a liquid-solid fluidized bed is the fluid-eddy convection rather than the particle carrier mechanism predominating in a gas-solid fluidized bed. Chiu and Ziegler (1985) have speculated that the controlling thermal resistance in a liquid-solid fluidized bed shifts from the region adjacent to the heater surface to the bed proper with a progressive decrease in either the particle size or superficial liquid velocity. They have further speculated that the maximum in the heat transfer coefficient in such a bed might result from the simultaneous effects of increasing resistance in the region adjacent to the heater surface and decreasing resistance in the bed proper. A surface renewal model has been developed by Deckwer (1980) based on the isotropic turbulence theory for heat transfer in bubble columns, and a model has been proposed by Lewis et al. (1982) based on the assumption that the heat is transferred through unsteady-state diffusion from the liquid layer to packets of liquid in the two-phase mixture in a bubble column. Recently, Muroyama et al. (1986) examined in detail the resistance to heat transfer from the wall-to-bed in a liquid-solid fluidized bed. They have pointed out that the effective thermal conductivity reflects directly the intensity of radial liquid mixing within the bed and that it attains a maximum as the liquid velocity or bed porosity is varied.

In the present study, heat transfer characteristics of a liquid-solid fluidized bed with an immersion heater were experimentally investigated. Specifically, the axial dispersion coefficients of fluidized particles were measured by means of a relaxation method to examine the effects of particle motion on the rate of heat transfer. Furthermore, the effects of the liquid flow rate or bed porosity and of the particle size on the heat transfer resistance in the region adjacent to the heater surface and that in the bed proper were examined. The resultant data have been analyzed in terms of the two resistance-in-series model.

Experiment

Experiments were carried out in a liquid-solid fluidized bed comprising a glass column, 3 m in height and 0.152 m in diameter. A detailed description of this experimental system can be found elsewhere (Kang et al., 1983, 1985).

As a heating source, a cone-shaped heater with an outside diameter (OD) of 0.030 m and a length of 0.356 m was placed coaxially on the distributor plate at the center of the column. Five iron-constantan thermocouples were installed radially 0.20 m above the distributor plate to measure the radial temperature profile in the bed. The location of the first thermocouple was at 5×10^{-3} m from the immersed heater surface; the second through fifth thermocouples were placed at the radial intervals of 0.005, 0.010, 0.020 and 0.021 m, respectively. These thermocouples were connected to a digital thermometer to record

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simultaneously and continuously all point temperatures. To record the pressure fluctuation histogram, a semiconductor type pressure transducer (Copal Electronics) was installed 0.20 m above the liquid distributor plate on the opposite side where the thermocouple was placed. The pressure transducer was connected to the recorder via an amplifier to determine the path of a pressure variation from one steady state to another with time. Glass beads with a diameter of 1.7×10^{-3} or 4.0×10^{-3} m and a density of 2,500 kg/m^3 served as the fluidized particles; water served as the fluidizing liquid.

A step change in the fluidizing condition in the bed was effected by manipulating the superficial liquid-flow rate. During the passage from one steady-state fluidizing condition to another, the pressure drop in the bed was measured as a function of time. When a new steady state was reached, the temperatures were measured repeatedly. The heat transfer coefficient was calculated as

$$h=\frac{q}{A\,(T_h-T_m)} \tag{1}$$

where q is the heat flux from the heater surface to the bed; A, the effective surface area of the heater; and T_h and T_m, the temperature of the heater surface and the mean temperature of the bed proper, respectively.

The heat flux, q, was obtained from the DC power supply, and the temperature difference between the immersed heater and bed was determined by

$$T_h-T_m=\frac{\int_o^R U(r)[T_h-T(r)]r\,dr}{\int_o^R U(r)\,r\,dr} \tag{2}$$

The expression for the radial liquid velocity distribution was assumed to be identical to that proposed for the fully developed flow in a fluidized bed (Morooka et al., 1982). The heat flux, q, was verified from the energy balance

$$q=\dot{m}\,C_{p\ell}(T_{mo}-T_{mi}) \tag{3}$$

where T_{mo} and T_{mi} are the outlet and inlet cross-sectional average temperatures, respectively.

Results and Discussion

The radial temperature profile in the region adjacent to the heater surface is much steeper than that in the bed proper, apparently indicating that two resistances connected in series, i.e., the resistance in the region adjacent to the heater surface and that in the bed proper, can be identified in a liquid-solid fluidized bed (Kang and Kim, 1987). Thus, heat transfer between the coaxially mounted immersed heater and liquid-solid fluidized bed can be represented by a two resistance-in-series model as in the case of a liquid-solid fluidized bed with heat transfer through its walls (Wasmund and Smith, 1967; Patel and Simpson, 1977; Muroyama et al., 1986). The actual temperature differences between the heater surface and fluidized bed ranged between 2.75 and 3.89°C.

Heat transfer resistance and boundary layer thickness around the heater

The overall heat transfer coefficient can be evaluated by combining the heat transfer coefficient in the region adjacent to the heater surface and that in the bed proper through a series relationship,

$$\frac{1}{h}=\frac{1}{h_h}+\frac{1}{h_b} \tag{4}$$

where h, h_h, and h_b are the overall heat transfer coefficient, the heat transfer coefficient in the region adjacent to the heater surface, and that in the bed proper, respectively.

The heat transfer coefficient in the region adjacent to the heater surface is obtained from the heat balance around the heater surface, thus giving

$$h_h=\frac{q}{(T_h-T_\delta)\,A} \tag{5}$$

where T_h and T_δ are the temperatures of the heater surface and boundary layer around it, respectively. The boundary layer temperature can be determined by extrapolating the radial temperature profile to the heater surface; the boundary layer is essentially a very thin liquid film around the heater (Wasmund and Smith, 1967; Patel and Simpson, 1977; Muroyama et al., 1986). It is plausible that the steady-state heat conduction takes place in the boundary layer, which is very thin relative to the bed radius (Lewis et al., 1982; McCabe et al., 1985). Thus, the heat transfer coefficient in the region adjacent to the heater surface can also be written as

$$h_h=\frac{k_\ell}{\delta} \tag{6}$$

where k_ℓ and δ are the thermal conductivity of the liquid phase and the boundary layer thickness around the heater, respectively. After determining h_h from Eq. 5 with the knowledge of T_h and T_δ, the value of the boundary layer thickness around the heater, δ, can be calculated from Eq. 6. The resultant values of h_h and δ are shown as functions of the liquid flow rate in Figures 1a and 1b, respectively. The values of the heat transfer resistance in the fluidized-bed proper, $1/h_b$, can be evaluated from Eq. 4 with the known values of h and h_h.

Particle dispersion

To examine the influences of fluidized particles on the mode and rate of heat transfer in liquid-solid fluidized beds, the particle motion and dispersion were determined by means of the relaxation method derived from the stochastic model of Yutani et al. (1982). When the liquid flow rate is increased stepwise at a steady-state fluidizing condition, the bed expands and reaches a new steady state corresponding to the new liquid flow rate. The new steady state is independent of the original liquid flow rate or fluidizing condition.

Note that the initially heterogeneous, unsteady expansion undergoes a transition to the homogeneous unsteady expansion at the relaxation point. In other words, the relaxation point corresponds to the onset of relaxation or the onset of homogeneous bed expansion. During the homogeneous expan-

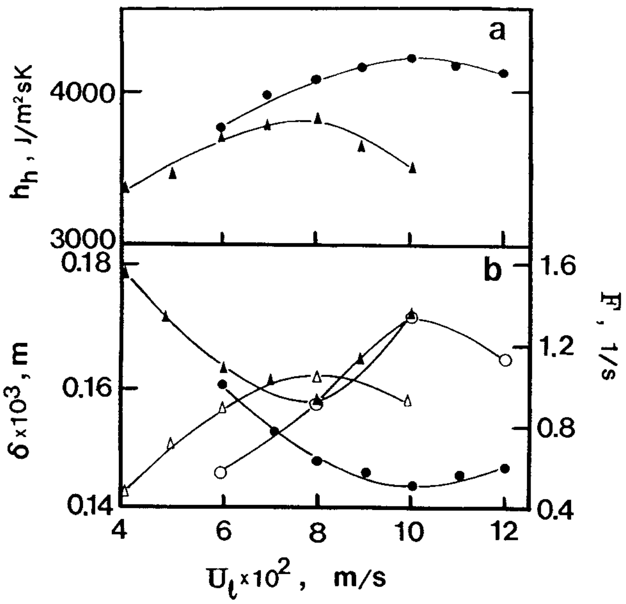

Figure 1. Effects of U_ℓ on h_h, δ and F in liquid-solid fluidized beds.

Key	$d_p\times10^3$(m)	Comment
▲	1.7	For h_h and δ values
●	4.0	
△	1.7	For F value
○	4.0	

sion, the mean fluctuating frequency of the particles, F, can be related to the solid particle holdup or bed porosity as (see, e.g., Yutani et al., 1982)

$$\frac{\epsilon_{\ell o}-\epsilon_{\ell a}(t)}{\epsilon_{\ell o}-\epsilon_{\ell R}}=\exp(-Ft) \tag{7}$$

The values of F have been recovered by fitting this expression to the experimentally obtained temporal changes of the bed porosity.

The axial dispersion coefficient of fluidized particles, D_p, and its variance, $\sigma_a^2(L_i)$, have been determined according to the approach proposed by Yutani et al. (1982) based on the concept of Bendat and Piersol (1971). The axial particle dispersion coefficient, D_p, can be expressed in terms of the solid particle holdup in the bed as follows:

$$D_p=\frac{F}{2}\ell^2\frac{\epsilon_{so}}{\epsilon_{SR}}\left(1-\frac{\epsilon_{so}}{\epsilon_{SR}}\right) \tag{8}$$

Experimentally determined F is shown in Figure 1b, and experimentally determined D_p in Figures 2 and 3a. The value of F in Figure 1b passes through a maximum as the liquid flow rate is increased from the minimum fluidization velocity in the bed of glass beads with a diameter of 1.7×10^{-3} or 4.0×10^{-3}m. This implies that the fluctuating frequency of a particle in the bed may vary with the liquid flow rate; Figures 2 and 3a demonstrate that the axial particle dispersion coefficients, D_p's, exhibit maxima with respect to the bed porosity, ϵ_ℓ, and the liquid flow rate, U_ℓ, respectively. Similar trends can be found in the results of previous investigators (Handley et al., 1966;

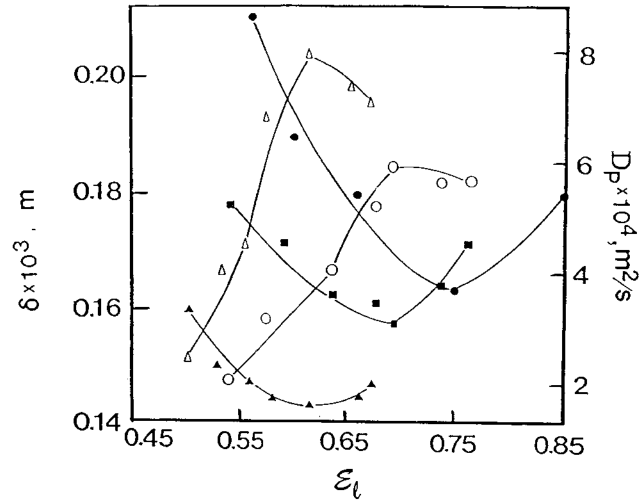

Figure 2. Effects of ϵ_ℓ on δ and D_p in liquid-solid fluidized beds.

Key	$d_p\times10^3$(m)	Source	Comment
■	1.7	This study	For δ value
▲	4.0	This study	
●	1.09	Wasmund & Smith (1976)	
○	1.7	This study	For D_p value
△	4.0		

Al-Dibouni and Garside, 1979; Yutani et al., 1982). Kang and Kim (1988) have pointed out that the flow regime transition of fluidized particles would occur in a liquid-solid fluidized

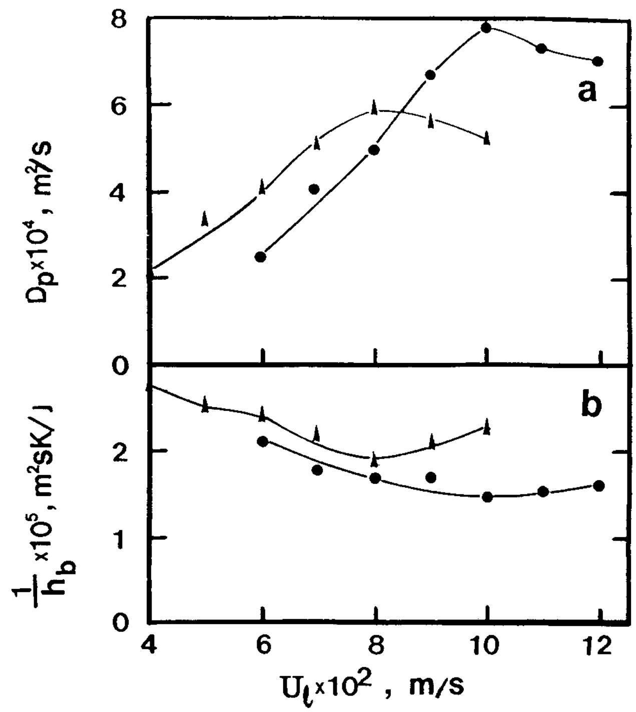

Figure 3. Effects of U_ℓ on D_p and $1/h_b$ in liquid-solid fluidized beds.

Key	$d_p\times10^3$(m)
▲	1.7
●	4.0

bed with increasing liquid flow rate or bed porosity. These values of the axial particle dispersion coefficient, D_p, are in good agreement with the correlation obtained by Van Der Meer et al. (1984).

Heat transfer in the region adjacent to the heater surface

Note that in Figures 1a and 1b the heat transfer coefficient in the region adjacent to the heater surface, h_h, exhibits a maximum and the thermal boundary layer thickness around the heater, δ, exhibits a minimum as the liquid flow rate, U_ℓ, is varied. Naturally, the δ exhibits a minimum value as the bed porosity in the liquid-solid fluidized bed varies (Figure 2). These figures demonstrate that the liquid flow rate or bed porosity at which the boundary layer thickness around the heater attains a minimum coincides closely with the bed porosity or liquid flow rate where the mean particle fluctuating frequency, F, or the axial particle dispersion coefficient, D_p, attains a maximum. It is reasonable, therefore, to state that the boundary layer thickness around the heater, δ, is minimum when the mean particle fluctuating frequency, F, or the particle dispersion coefficient, D_p, is maximum. The increase in the liquid flow rate or bed porosity may enhance the mobility of particles in the vicinity of the heater, which may result in a higher particle fluctuating frequency or higher contacting frequency between the particles and heater surface, and, consequently, in the erosion of the thermal boundary layer around the heater. As the liquid flow rate or, equivalently, bed porosity is further increased, the particle fluctuating frequency or contacting frequency of solid particles with the heater surface may be reduced due to the substantial decrease in the solid particle concentration or holdup in the bed (Kang et al., 1985).

The reduction in the solid particle holdup also causes the decrease in the intensity of turbulence in the bed generated by the fluidized particles (Joshi, 1983). Thus, the boundary layer thickness around the heater, δ, increases with the increase in the liquid flow rate or bed porosity. A similar trend has been observed by Richardson and Mitson (1958) and Wasmund and Smith (1967) in liquid fluidized beds. Recently, Muroyama et al. (1986) have found that the effective radial thermal conductivity and heat transfer coefficient, as functions of the liquid flow rate, attain maxima in a liquid-solid fluidized bed with wall-to-bed heat transfer. According to Kang and Kim (1988), the movement of fluidized solid particles may shift from the circulation mode to the turbulent or random mode at the bed porosity where the energy dissipation rate is maximum.

Figure 2 shows that the boundary layer thickness for a bed with larger particles was thinner than that for a bed with smaller particles under comparable conditions; thus, it is highly plausible that the larger particles are more effective in eroding the thermal boundary layer. It is generally known that the heat transfer coefficient in the liquid-solid fluidized bed is greater for the larger fluidized particles than for the smaller fluidized particles.

Heat transfer in the bed proper

The heat transfer resistance in the bed proper, $1/h_b$, can be estimated from Eq. 4, based on the two resistance-in-series models. The resultant values of $1/h_b$ are plotted against the liquid flow rate, U_ℓ, in Figure 3b, which exhibits a minimum.

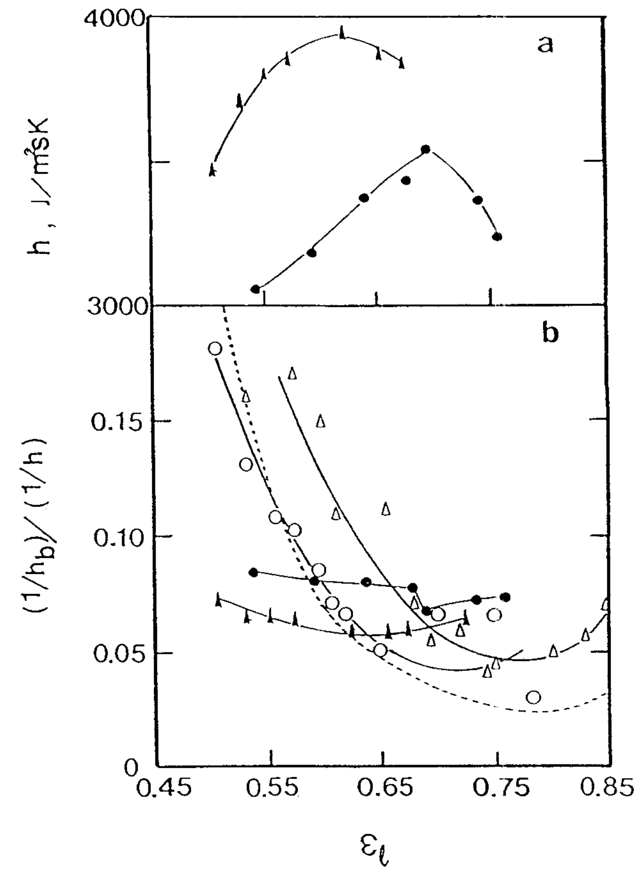

Figure 4. Effects of ϵ_ℓ on h and $(1/h_b)/(1/h)$ in liquid-solid fluidized beds.

Key	$d_p \times 10^3$(m)	Particle	Source
●	1.7	glass bead	This study
▲	4.0		
○	6.9	glass bead	Muroyama et al.
△	2.0	alumina bead	(1986)
---	0.58	lead sphere	Simpson (1973)

The ratio of the bed proper resistance to the overall resistance was less than 10% under the present experimental conditions, as can be seen in Figures 4a and 4b. For comparison, the experimental results of Simpson (1973) and Muroyama et al. (1986), obtained in the wall-to-bed heat transfer systems, are also plotted in Figure 4b. Note that their values of the ratio

$$\frac{1/h_b}{1/h}$$

are somewhat higher than those of the present study when the bed porosities are relatively low (less than 0.6). This may be attributed to the differences in the radial liquid dispersion coefficients.

It has been reported that the radial liquid dispersion coefficient increases with an increase in the diameter of a liquid-solid fluidized bed (Kang and Kim, 1986; Yasunishi et al., 1987). The diameter of the bed employed by Muroyama et al. (1986) was 0.0986 m and that by Simpson (1973) was only 0.0508 m; the diameter of the bed of the present study was 0.152 m. Thus, the extent of radial liquid dispersion in the bed of the present study was probably greater than that of

Muroyama et al. (1986) or Simpson (1973) when the bed porosities were low; this might have led to the decreases in the heat transfer resistance in the bed proper.

Correlation

Under fully fluidized conditions, the heat transfer resistance in the region adjacent to the heater surface contributes significantly to the overall heat transfer resistance; in contrast, the resistance in the bed proper is insignificant. The heat transfer coefficient in the region adjacent to the heater surface depends strongly on the liquid flow rate or bed porosity; therefore, h_h has been correlated in terms of the Stanton number containing the liquid flow rate and bed porosity. According to Wasmund and Smith (1967), Patel and Simpson (1977), and Kang and Kim (1987), the semilogarithmic plot of the Stanton number based on the heat transfer coefficient in the region adjacent to the heater surface decreases linearly with increasing bed porosity in liquid-solid fluidized beds. Chiu and Ziegler (1983) have correlated the modified Colburn *j*-factor, j_H, for heat transfer in such beds as a function of the modified particle Reynolds number as given below.

$$j_H = St_M P_r^{2/3} \phi_s = 0.1234\, Re_M^{-0.333} \tag{9}$$

This correlation is based on the overall heat transfer coefficient for wall-to-bed heat transfer. On the other hand, Muroyama et al. (1986) have correlated the wall-to-bed heat transfer coefficient in the region adjacent to the heater surface; specifically, the modified Colburn *j*-factor, $j_{H,h}$, is expressed as a function of the modified particle Reynolds number, Re_M, as follows:

$$j_{H,h} = 0.137\, Re_M^{-0.271} \tag{10}$$

The values of the modified *j*-factor based on h_h obtained in the present study have also been correlated as a function of the modified particle Reynolds number; it is expressed as

$$j_{H,h} = 0.191\, Re_M^{-0.310} \tag{11}$$

The correlation coefficient is 0.988.

In Figure 5, the present data are compared with those of the previous investigators (Wasmund, 1966; Simpson, 1973; Chiu and Ziegler, 1983; Muroyama et al., 1986). The figure reveals a generally good fit of Eq. 11 with the results of the previous investigators. The data obtained by Chiu and Ziegler (1983) are somewhat lower than those obtained by the others; Chiu and Ziegler have adopted j_H rather than $j_{H,h}$.

The dispersion or mixing of fluidized particles possibly can have an appreciable effect on both the rate of heat transfer in the region adjacent to the heater surface and that in the bed proper. Thus, an expression has also been developed in terms of the Colburn *j*-factor based on the overall heat transfer coefficient, j_H, and the modified Peclet number based on the axial dispersion coefficient of the particles, Pe_M; the resultant expression is

$$j_H = St_M Pr^{2/3} = 0.021\, Pe_M^{-0.453} \tag{12}$$

The effects of the particle dispersion and fluctuations on the heat transfer resistance in the region adjacent to the heater surface and that in the bed proper probably manifest themselves in the magnitude of the boundary layer thickness around the heater and in the intensity of the turbulence in the bed, respectively. Equation 12 indicates that an increase in the particle dispersion coefficient, D_p, leads to an increase in the overall heat transfer coefficient, h. A comparison of the calculated and experimentally measured j_H's yields a regression coefficient of 0.954, thereby indicating that they are in sufficiently good accord.

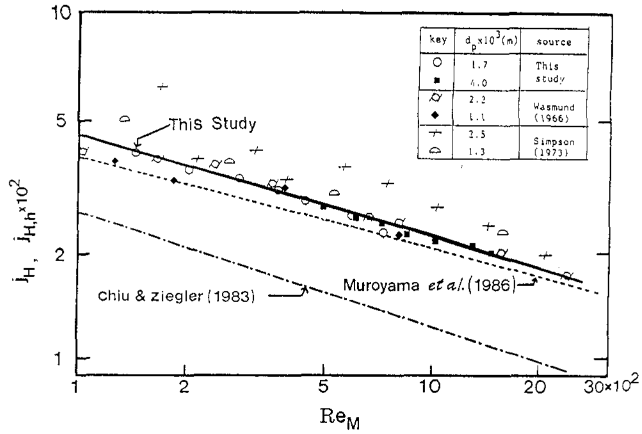

Figure 5. j_H and $j_{H,h}$ vs. Re_M in liquid-solid fluidized beds.

Notation

A = heater surface area, m^2
$c_{p\ell}$ = specific heat of the liquid phase, J/kg·K
D = column diameter, m
D_p = particle dispersion coefficient, m^2/s
d_p = particle size, m
F = mean fluctuating frequency of the particles, 1/s
h = overall heat transfer coefficient, $J/m^2 \cdot s \cdot K$
h_b = heat transfer coefficient in the bed proper, $J/m^2 \cdot s \cdot K$
h_h = heat transfer coefficient in the region adjacent to the heater, $J/m^2 \cdot s \cdot K$
j_H = modified Colburn *j*-factor, $(h/\rho_\ell c_{p\ell} U_\ell)\epsilon_\ell Pr^{2/3}$
$j_{H,h}$ = modified Colburn *j*-factor in the region adjacent to the heater, $(h_h/\rho_\ell c_{p\ell} U_\ell)\epsilon_\ell Pr^{2/3}$
k_ℓ = thermal conductivity of the liquid phase, J/m·s·K
ℓ = position of the pressure sensor, m
L_i = location of the individual particle, m
m = mass flow rate of the liquid phase, kg/s
Pe_M = modified Peclet number, $d_p U_\ell \epsilon_\ell / D_p(1-\epsilon_\ell)$
Pr = Prandtl number, $c_{p\ell}\mu_\ell/k_\ell$
q = heat flow rate, J/s
Re_M = modified Reynolds number, $d_p U_\ell \rho_\ell / \mu_\ell(1-\epsilon_\ell)$
St_M = modified Stanton number, $(h/\rho_\ell c_{p\ell} U_\ell)\epsilon_\ell$
T = temperature, K
$U(r)$ = radial liquid velocity profile, m/s
U_ℓ = superficial liquid velocity, m/s

Greek letters

δ = average boundary layer thickness, m
ϵ = phase holdup
μ = viscosity, Pa·s
ρ = density, kg/m^3
σ^2 = variance
ϕ = shape factor

Subscript

a = after relaxation
b = bed proper

h = heater surface or region adjacent to it
ℓ = liquid phase
m = mean value
o = steady state
R = relaxation point
s = solid phase
δ = boundary layer around the heater

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